The Falling Slinky

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Introduction

•The Slinky was invented by Richard James, a naval engineer, and his wife Betty James in 1943

•It was originally meant to stabilize naval instruments

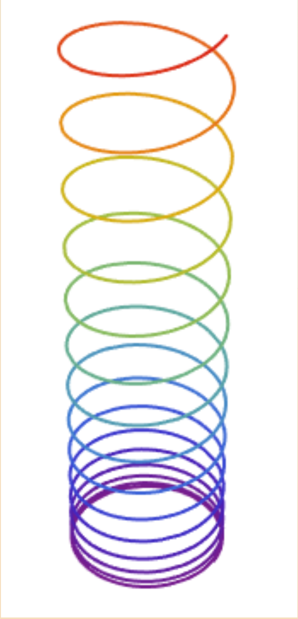
•Slinkies have been used as radio antennas in the Vietnam War as well as in zero gravity tests by NASA

•Since the toy was first sold in 1945, over 300 million have been sold worldwide

•The question of 'how does a slinky fall?' was first modeled by M. G. Calkin and later popularized by Martin Gardner in his monthly column "Mathematical Games" in Scientific American. It goes as follows:

If I were to drop a slinky vertically, what would happen right after you let go of the top?

•The top end would fall first
•The bottom end would fall first
•Both ends would fall at the same time
•The top and bottom ends would approach each other with the center of mass at rest



Background Knowledge

• Hooke's law: F = k * x
Where F is force, k is the spring constant, and x is the elongation or compression of the spring

• A slinky is a tension spring
  • Its resting position is fully compressed and therefore it can only elongate

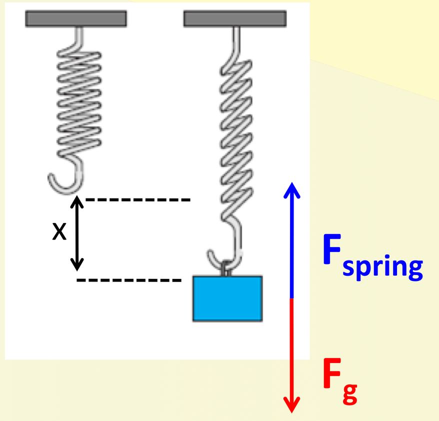
• Newton's Second Law: F = m * a
Where F is force, m is mass, and a is acceleration

$F_{net} = m * 0$

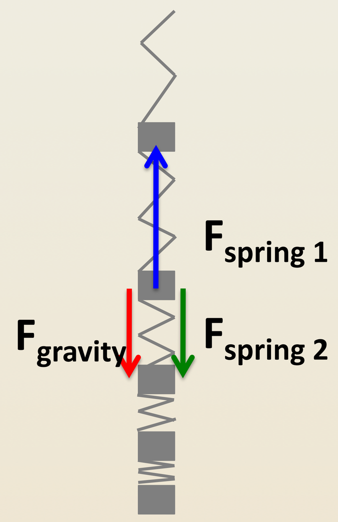
$F_{net} = 0 = F_g + F_{spring}$

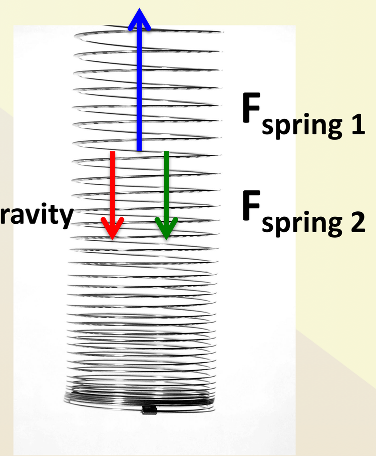
$F_{net} = 0 = mg + kx$

$-kx = mg$

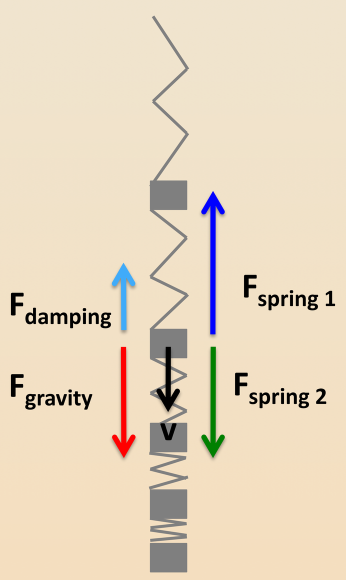


Forces on an unmoving spring





Forces on a moving spring



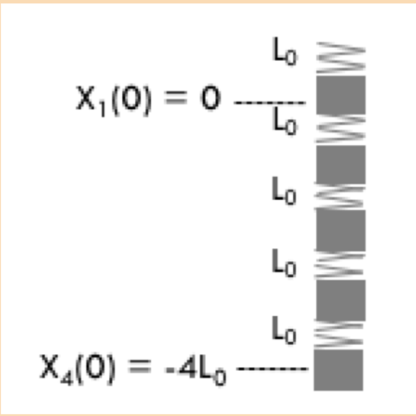
$F_{net} = F_g + F_{spring\ 1} + F_{spring\ 2} + F_{damping}$

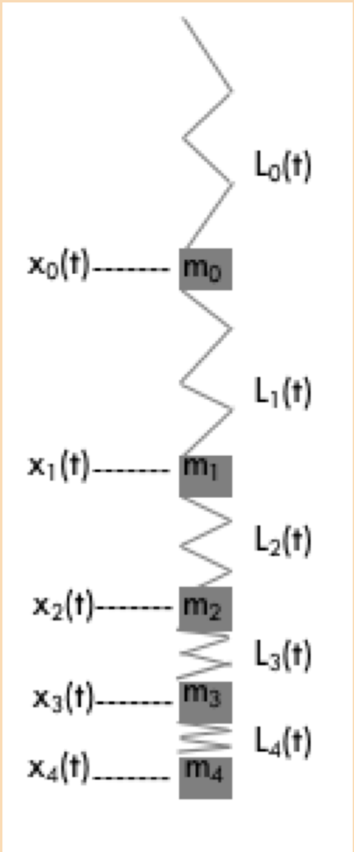
$F_{net} = mg + kx_1 - kx_2 + cv$

$F_{damping} = cv$

Discrete Model

•Imagine N coils represented as masses connected by massless springs, all with the same rest-length, like so:





•For example, $x_2(t)$ is the position of mass 2 at time t.

• $L_2(t)$ is the length of the spring directly above mass 2 at time t

•Generally, the lengths of the springs and the forces acting on each mass can be found through the position:

For N number of masses: J=1, 2, 3, …, N

$L_J = X_{(J-1)} - X_J$

if J<N,

$F_J = mg + K_J (L_J - L_0) - K_{(J+1)} (L_{J+1} - L_0)$

if J=N,

$F_J = mg + K_J (L_J - L_0)$

The method is as follows

1. Position
2. Spring Length
3. Forces
4. Forces + previous velocity = acceleration
5. Position + previous velocity = next position
6. Acceleration + previous velocity = new velocity

Repeat steps for each mass, for many increments of time dt.

Computer Model

A computer code, written by Michael de Podesta at the UK National Physics lab, analyzes each mass in this model 2000 times following the method previously stated, calculating the elongation of each connecting spring over the course of 4 seconds.

•Number of masses: N = 10 masses

•Mass of a single mass: m = 1 kg

•Time increment: dt = 0.003 seconds

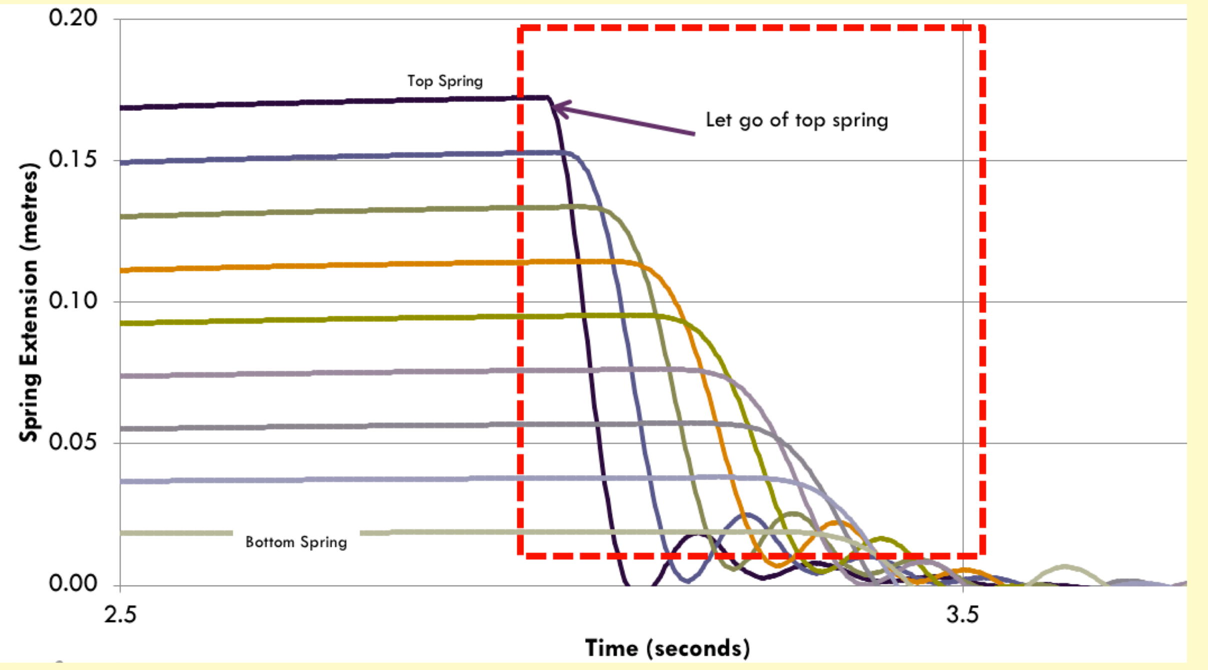
•Acceleration of gravity: g = -9.8 N/kg

•Initial (resting) length of the springs: L_0 = 0.3 meters

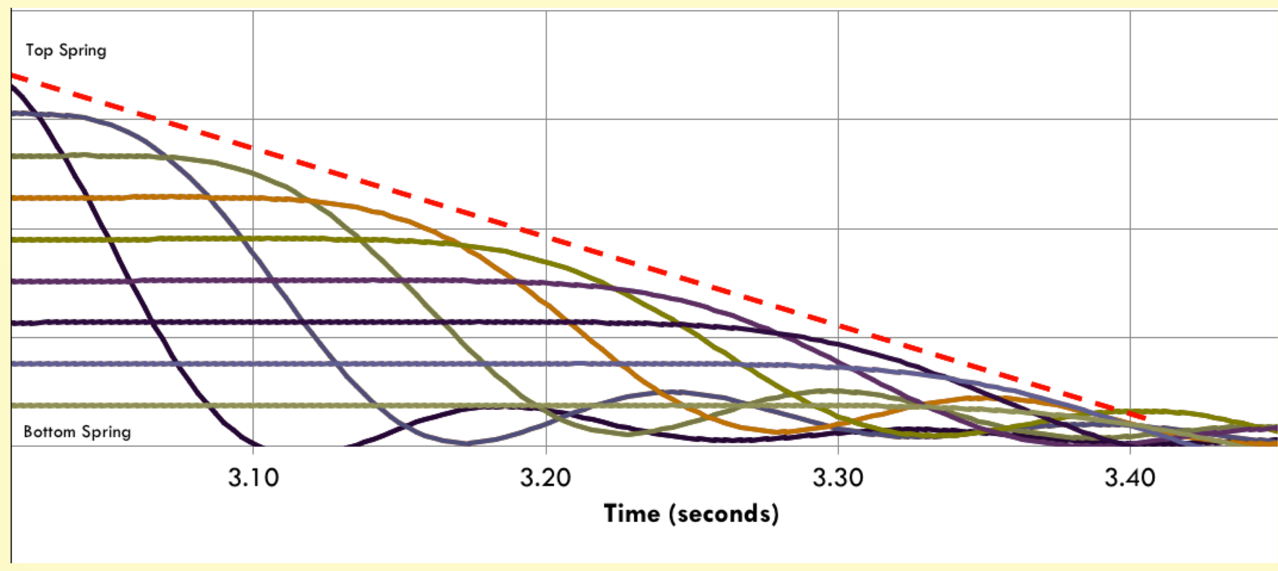
•Spring Constant: k = 500 N/m

•Damping Coefficient: c = 10

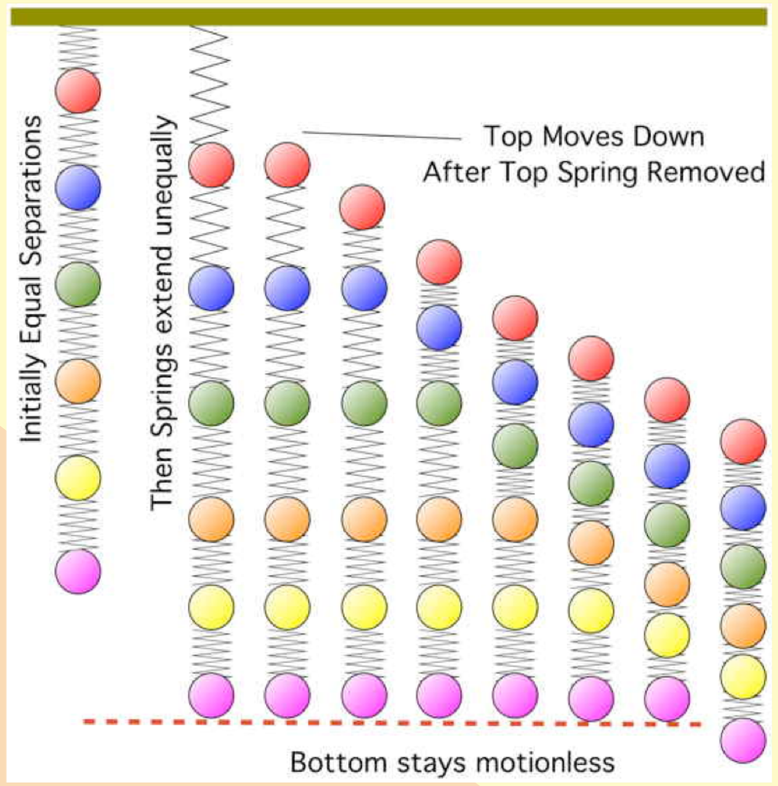
Results



Area within the red dotted line:



The elongation results generated by the computer code are graphed above. The results show that the spring elongations do not change until the spring directly above it begins to collapse. Visually, this results in the 'levitating slinky effect,' where the bottom of the slinky remains in its starting location until the slinky components above it have collapsed. (I suggest looking up falling slinky on Youtube for a more impressive visualization). The dotted red line in the second graph represents the speed of the propagating longitudinal wave as it travels down the slinky.



References

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4. https://www.insidescience.org/news/secrets-levitating-slinky